SOME PROBLEMS IN THE DYNAMICS OF MAGNETO-VORTEX CONFIGURATIONS

(NEKOTORYE ZADACHI DINAMIKI MAGNETNO-VIKHREVYKH KONFIGURATSII)

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Iu.P. LADIKOV
(Moscow)

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'he equations of motion of a system of coaxial magneto-vortex rings are leveloped and some particular cases of this type of motion are investigated. By magneto-vortex rings is meant circular, vortical plasma filaments along which currents flow. Outside the rings the fluid is taken to be ideal: incompressible and nonconducting. The conditions for the existence and stability of an isolated magneto-vortex ring were investigated in [1]. The analog of a system of coaxial rings is a pair of rectilinear magneto-vortex filaments with mutually opposed circulations and currents, arranged symmetrically with respect to a given axis. It is assumed that the motion of a magneto-vortex ring does not differ qualitatively from the motion of the corresponding pair of rectilinear magneto-vortex filaments; the motion of such a pair in the direction of a conducting and a nonconducting wall is investigated. It is shown that in the first case the ring will approach the wall and expand. In the second case, for certain values of the initial parameters, the ring contracts, approaching the wall. If, in this case, there is an aperture in the wall, the ring can jump through it. Such phenomena, according to witnesses, are observed when ball lightning approaches an obstacle [2].

1. Fundamental equations. It is known that the motion of bodies with multiconnected volumes in an incompressible fluid may be described by the system of equations

$$\frac{d}{dt}\frac{\partial T}{\partial \dot{q}_i} - \frac{\partial T}{\partial q_i} + \sum_{j-1}^{n} \gamma_{ij}\dot{q}_j + \frac{\partial K}{\partial q_i} = Q_i \qquad (i = 1, \ldots, n)$$

$$T = \sum_{i,j} A_{ij}\,\dot{q}_i\dot{q}_j\,, \qquad K = \sum_{i,j} B_{ij}\Gamma_i\Gamma_j, \qquad \gamma_{rs} = \frac{\partial \beta_s}{\partial q_r} - \frac{\partial \beta_r}{\partial q_s} \qquad (1.1)$$

Here q_i are generalized coordinates, $\dot{q}_i$ are generalized velocities,

T is that part of the kinetic energy which the moving bodies and the fluid would have in the absence of cyclic motion; the quadratic form K of the circulations Γ_i represents the kinetic energy of the cyclic motion; Q_i is the generalized force corresponding to the generalized coordinate q_i; the quantities β_r are linear forms of the circulation, $\beta_r = a_{r1}\Gamma_1 + \ldots + a_{rk}\Gamma_k$, whose coefficients are determined from the equations

$$\rho\dot{\chi}_j = \frac{\partial K}{\partial \Gamma_j} + \sum_{i=1}^{n} \alpha_{ij}\dot{q}_i \qquad (j = 1, \ldots, k) \quad (k \text{ — is the order of connectedness}) \tag{1.2}$$

If the region of flow under consideration is made singly connected by means of k partitions, then $\rho\dot{\chi}_j$ defines the mass flow per unit time across the partition j, and, therefore, a_{ij} is the portion of mass flow across partition j associated with unit time rate of change of the coordinate q_i. Equations (1.1) are seldom used, since the quadratic forms T and K are, generally speaking, unknown, and can be determined only from the solution of the corresponding hydrodynamic problem, which in the majority of cases is very difficult. However, if the bodies being considered are very thin, for instance vortex filaments, the equations can be applied.

Let us go to some concrete examples. Consider an ideal incompressible nonconducting fluid, containing thin plasma vortex rings or rectilinear filaments, along which currents I_j are flowing, having the same directions as the vortices or opposite to them. Outside the vortex filaments the fluid motion is irrotational and cyclic, with a velocity which is determined by the Biot-Savart formula. The currents flowing along the filaments induce in the surroundings a magnetic field, whose intensity is also defined by the Biot-Savart law. It is considered that the field does not interact with the surrounding medium. In view of this, the circulation Γ around an arbitrary closed contour must be conserved, and, consequently, the vortex filaments cannot leave that portion of the plasma to which they became attached in the initial moment of motion; that is, the vortex filaments will be frozen in the corresponding plasma filaments. Since the rings are assumed to be very thin, and the quadratic form T is proportional to the volumes τ of the rings, we may neglect the quantity T and its derivatives in Equations (1.1).

The generalized forces Q_i in the case under consideration are determined [4] as derivatives of the magnetic energy M of the currents, taken in the directions of the corresponding coordinates $Q_i = \partial M/\partial q_i$. Equations (1.1) take the following form:

$$\sum \gamma_{ij}\dot{q}_j + \frac{\partial K}{\partial q_i} = \frac{\partial M}{\partial q_i} \qquad (i = 1, \ldots, n) \tag{1.3}$$

In addition, considering the plasma rings and filaments to be ideal conductors, we can add to the system (1.3) the conditions for constant flux of magnetic induction across the surface bounding the contour of the conductor

$$\frac{d}{dt}\frac{\partial M}{\partial I_j} = 0 \tag{1.4}$$

Thus the plasma rings and filaments must satisfy the system of equations (1.3) and (1.4).

Let us first investigate the motion of one plasma ring. We define the position of the ring by its radius $r = q_i$, the radius of its cross-section μ, the coordinates ($\xi = q_2$, $\nu = q_4$) of the point of intersection of the plane of the ring with the axis of symmetry, and the angle $\theta = q_3$, which the axis makes with a chosen direction (Fig. 1). The kinetic energy of the motion of the fluid due to a single vortex ring and the magnetic energy of the circular current are, respectively

$$K = \frac{\rho\Gamma^2 r}{2}\left(\ln\frac{8r}{\mu} - \frac{7}{4}\right), \qquad M = \frac{2\pi r}{c^2} I^2\left(\ln\frac{8r}{\mu} - \frac{7}{4}\right)$$

Here ρ is the density, c the velocity of light in vacuum. It should be noted that r and μ are connected by the condition of incompressibility, $2\pi^2 r\mu^2 = \tau = \text{const}$.

It is easy to see that of the quantities α_{rj}, which are equal to the mass flows through the hole in the ring, only

$$\alpha_{21} = -\pi r^2 \cos\theta\rho, \qquad \alpha_{41} = -\pi r^2 \sin\theta\rho$$

are different from zero. Correspondingly,

$$\beta_2 = -\Gamma\pi r^2\rho\cos\theta, \qquad \beta_4 = -\Gamma\pi r^2\rho\sin\theta$$

The remaining quantities $\beta_i = 0$. The equations of motion of the ring will have the following form:

$$+\pi r^2\rho\cos\theta\,\dot{\theta}\Gamma + 2\pi r\rho\sin\theta\Gamma\dot{r} = 0$$

$$2\pi r\rho\cos\theta\Gamma\dot{r} - \pi r^2\rho\sin\theta\Gamma\dot{\theta} = 0$$

$$-2\pi r\rho\cos\theta\Gamma\dot{\xi}-2\pi r\rho\sin\theta\Gamma\dot{\eta}=-\frac{\partial K}{\partial r}+\frac{\partial M}{\partial r}\quad(1.5)$$

$$\pi r^2\rho\sin\theta\Gamma\dot{\xi}-\pi r^2\rho\cos\theta\Gamma\dot{\eta}=0$$

$$\frac{4\pi r}{c^2}I\left(\ln\frac{8r}{\mu}-\frac{7}{4}\right)=\Phi=\text{const}$$

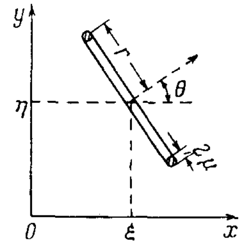

Fig. 1.

It is evident that the first two equations can be satisfied only for r = const, θ = const. Then, from the remaining equations, it follows that an isolated ring can only move forward in the direction of its axis, with a constant velocity

$$v=\frac{\Gamma}{4\pi r}\left(\ln\frac{8r}{\mu}-\frac{1}{4}\right)(1-\zeta)^2,\qquad\zeta=\sqrt{\frac{4\pi}{\rho}}\frac{I}{c\Gamma}$$

This formula agrees with the formula for the velocity of forward motion of a magneto-vortex ring as found from the solution of the exact equations of magnetohydrodynamics, under the assumption that the ring has small, but finite, dimensions [1]. At first glance, it seems paradoxical that the ring can have no motion other than in the direction of its axis of symmetry. In fact, suppose we gave the ring some other initial motion, for example, rotation. Then the ring should rotate under its inertia for at least some time. However, having neglected the mass (a measure of the inertia) in the development of the equations, it is not possible to consider that the ring can have inertia, and, consequently, it is not possible to impose an initial velocity. Equations (1.3)-(1.4) determine the velocities directly, not the accelerations.

Now let us investigate the motion of two magneto-vortex rings with a common axis of symmetry z. For the generalized coordinates we can take $q_1 = r_1$, the radius of the first ring; $q_2 = z_1$, the distance of its center from the origin of coordinates; $q_3 = r_2$, the radius of the second ring; $q_4 = z_2$, the distance of the center of the second ring from the origin. The kinetic energy K of the cyclic motion and the magnetic energy M are, respectively

$$K=\frac{\rho\Gamma_1^2r_1}{4}\left(\ln\frac{128\pi^2}{\tau_1}r_1^3-\frac{7}{2}\right)+\frac{\rho\Gamma_2^2r_2}{4}\left(\ln\frac{128\pi^2}{\tau_2}r_2^3-\frac{7}{2}\right)+2\rho\Gamma_1\Gamma_2\sqrt{r_1r_2}\,f(k)$$

$$M=\frac{I_1^22\pi r_1}{c^2}\left(\ln\frac{128\pi^2}{\tau_1}-\frac{7}{2}\right)-\frac{I_2^22\pi r_2}{c^2}\left(\ln\frac{128\pi^2}{\tau_2}-\frac{7}{2}\right)+\frac{I_1I_2}{c^2}4\pi\sqrt{r_1r_2}f(k)$$

Here τ_1, τ_2 are the volumes of the rings.

$$k^2 = \frac{4r_1r_2}{(r_1+r_2)^2+(z_1-z_2)^2}, \qquad k'^2 = 1-k^2, \qquad f(k) = \frac{1+k'^2}{2k}F - \frac{1}{k}E$$

where F and E are complete elliptic integrals. From Equations (1.2) it may be found that $a_{12} = -\pi r_1^2\rho$, $a_{24} = -\pi r_2^2\rho$, analogously to the results for the preceding example. All the remaining quantities $a_{ij} = 0$. Then, calculating the values β_i and γ_{ij}, and putting them in (1.3) and (1.4), we obtain

$$\frac{dr_1}{dt} = \frac{\Gamma_2}{\pi}(1-\zeta_1\zeta_2)\frac{k_3(z_1-z_2)}{4r_1}\frac{\partial f}{\partial k}$$

$$\frac{dr_2}{dt} = -\frac{\Gamma_1}{\pi}(1-\zeta_1\zeta_2)\frac{k_3(z_1-z_2)}{4r_2}\frac{\partial f}{\partial k}$$

$$\frac{dz_1}{dt} = (1-\zeta_1^2)\frac{3}{8\pi}\frac{\Gamma_1}{r_1}\ln\lambda_1 r_1 + \frac{\Gamma_2}{\pi}(1-\zeta_1\zeta_2)\left(\frac{r_2}{r_1}\right)^{\frac{1}{2}}\left[\frac{1}{2r_1}f(k) + \frac{\partial f}{\partial k}\frac{\partial k}{\partial r_1}\right] \tag{1.6}$$

$$\frac{dz_2}{dt} = (1-\zeta_2^2)\frac{3}{8\pi}\frac{\Gamma_2}{r_2}\ln\lambda_2 r_2 + \frac{\Gamma_1}{\pi}(1-\zeta_1\zeta_2)\left(\frac{r_1}{r_2}\right)^{\frac{1}{2}}\left[\frac{1}{2r_2}f(k) + \frac{\partial f}{\partial k}\frac{\partial k}{\partial r_2}\right]$$

$$2I_1r_1\left(\ln\frac{128\pi^2}{\tau_1}r_1^3 - \frac{7}{2}\right) + I_2\sqrt{r_1r_2}f(k) = \Phi_1 = \text{const}$$

$$2I_2r_2\left(\ln\frac{128\pi^2}{\tau_2}r_2^3 - \frac{7}{2}\right) + I_1\sqrt{r_1r_2}f(k) = \Phi_2 = \text{const}$$

Here

$$\zeta_1 = \sqrt{\frac{4\pi}{\rho}}\frac{I_1}{c\Gamma_1}, \quad \zeta_2 = \sqrt{\frac{4\pi}{\rho}}\frac{I_2}{c\Gamma_2}, \quad \lambda_1 = \left(\frac{128\pi^2}{\tau_1\sqrt{e}}\right)^{\frac{1}{3}}, \quad \lambda_2 = \left(\frac{128\pi^2}{\tau_2\sqrt{e}}\right)^{\frac{1}{3}}$$

For $M = 0$ the system of equations (1.6) becomes identical with the system of equations which define the motion of two coaxial vortex rings, which may be obtained from purely kinematic considerations [5]. It is easy to see that the system (1.6) has an integral of conservation of impulse, $\Gamma_1 r_1^2 + \Gamma_2 r_2^2 = \text{const}$, and an integral of energy conservation, $K + M = \text{const}$.

The first integral can be easily obtained from the first two equations. It has the following physical significance. From the general theorems of hydrodynamics it follows that any irrotational motion in a singly-connected region can be established by impulsive pressures applied to the boundaries of the region. If the rings are divided by partitions, the region of flow under consideration becomes singly-connected and the motion in it can be established by means of impulsive pressure $\pi r_1^2\Gamma_1$ and $\pi r_2^2\Gamma_2$ applied to the partitions. Due to the absence of external forces the sum of these pressures must remain constant.

The second integral can be obtained by computing the total derivative

$$\frac{d}{dt}(K-M)=\sum_{i=1}^{4}\frac{\partial}{\partial q_i}(K-M)\frac{dq_i}{dt}+\sum_{j=1}^{2}\frac{\partial(K-M)}{\partial I_j}\frac{dI_j}{dt} \tag{1.7}$$

The first sum is zero, in view of the first four equations of (1.6), and the second sum can be transformed in the following way:

$$\sum_{j=1}^{2}\frac{\partial(K-M)}{\partial I_j}\frac{\partial I_j}{dt}=-\frac{d}{dt}\sum_{j=1}^{2}I_j\frac{\partial M}{\partial I_j}+\sum_{j=1}^{2}I_j\frac{d}{dt}\frac{\partial M}{\partial I_j}=-2\frac{dM}{dt}$$

Putting this into (1.7) we obtain $K+M=$ const.

The system of equations (1.6) is very difficult, and therefore, instead of the motion of coaxial magneto-vortex rings, it is expedient to investigate the motion of a magneto-vortex pair with a common axis. By magneto-vortex pair we mean the combination of two rectilinear, plasma vortex filaments, along which equal currents are flowing in opposite directions, and which have equal and opposite circulations.

If the rings are very thin, it may be assumed that the motion of magneto-vortex rings will not differ qualitatively from the motion of the corresponding magneto-vortex pairs, as is true also in the pure hydrodynamic case. The kinetic energy of a system of two magneto-vortex pairs with a common axis y has the following value:

$$K=\frac{\rho\Gamma_1^2}{8\pi}\left(1+4\ln\frac{2x_1}{\mu_1}\right)+\frac{\rho\Gamma_2^2}{8\pi}\left(1+4\ln\frac{2x_2}{\mu_2}\right)+\frac{\rho\Gamma_1\Gamma_2}{2\pi}F(x_1,x_2,y_1,y_2)$$

The magnetic energy is

$$M=\frac{I_1^2}{2c^2}\left(1+4\ln\frac{2x_1}{\mu_1}\right)+\frac{I_2^2}{2c^2}\left(1+4\ln\frac{2x_2}{\mu_2}\right)+\frac{2I_1I_2}{c^2}F(x_1,x_2,y_1,y_2)$$

$$F(x_1,x_2,y_1,y_2)=-\ln\frac{(x_1-x_2)^2+(y_1-y_2)^2}{(x_1+x_2)^2+(y_1-y_2)^2}$$

Here $2x_1$, $2x_2$ are the distances between the vortices of the two pairs; y_1, y_2 are the ordinates of the pairs. Computing, as in the preceding, the expressions for α_{ij}, β_i, γ_{ij} and putting them into (1.3) and (1.4) we obtain

$$\frac{dx_1}{dt} = -(1-\zeta_1\zeta_2)\frac{\Gamma_2}{4\pi}\frac{\partial F}{\partial y_1}, \qquad \frac{dx_2}{dt} = -(1-\zeta_1\zeta_2)\frac{\Gamma_1}{4\pi}\frac{\partial F}{\partial y_2}$$

$$\frac{dy_1}{dt} = (1-\zeta_1^2)\frac{\Gamma_1}{4\pi x_1} + \frac{\Gamma_2}{4\pi}(1-\zeta_1\zeta_2)\frac{\partial F}{\partial x_1} \tag{1.8}$$

$$\frac{dy_2}{dt} = (1-\zeta_2^2)\frac{\Gamma_2}{4\pi}\frac{1}{x_2} + \frac{\Gamma_1}{4\pi}(1-\zeta_1\zeta_2)\frac{\partial F}{\partial x_2}$$

$$2\Gamma_1\zeta_1 \ln \lambda_1 x_1 + \Gamma_2\zeta_2 F(x_1, x_2, y_1, y_2) = \Phi_1 = \text{const}$$

$$2\Gamma_2\zeta_2 \ln \lambda_2 x_2 + \Gamma_1\zeta_1 F(x_1, x_2, y_1, y_2) = \Phi_2 = \text{const}$$

Here μ_1, μ_2 are the radii of the sections of the filaments

$$\zeta_j = \sqrt{\frac{4\pi}{\rho}}\frac{I_j}{c\Gamma_j} \quad (j=1,2), \qquad \lambda_j = \frac{\sqrt[4]{e}}{2\mu_j} \quad (j=1,2)$$

These equations, like the preceding ones, have integrals of conservation of impulse and energy

$$\Gamma_1 x_1 + \Gamma_2 x_2 = 2c = \text{const}$$

$$(1+\zeta_1^2)\frac{\Gamma_1^2}{2\pi}\ln\lambda_1 x_1 + (1+\zeta_2^2)\frac{\Gamma_2^2}{2\pi}\ln\lambda_2 x_2 - \tag{1.9}$$

$$-(1+\zeta_1,\zeta_2)\frac{\Gamma_1\Gamma_2}{2\pi}\ln\frac{(x_1-x_2)^2+(y_1-y_2)^2}{(x_1+x_2)^2+(y_1-y_2)^2} = \text{const}$$

As may be seen from the equations, one magneto-vortex pair moves in the direction of its axis y with velocity

$$U = \frac{\Gamma}{4\pi x}(1-\zeta^2) \qquad \left(\zeta = \frac{\Phi}{2\Gamma\ln\lambda x} = \text{const}\right)$$

2. Motion of a magneto-vortex pair in the presence of a conducting wall. Let us investigate the motion of a magneto-vortex pair in the direction of an infinitely conducting wall. The problem of this type of motion is equivalent to the problem of the motion of two magneto-vortex pairs with equal values of the abscissa, $x_1 = x_2 = x$, and equal but opposite values of ordinate, circulation and current

$$y_1 = -y_2 = y, \quad \Gamma_1 = -\Gamma_2 = \Gamma_1$$
$$I_1 = -I_2 = I, \quad \zeta_1 = \zeta_2 = \zeta$$

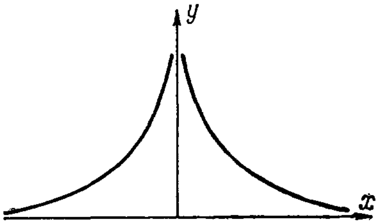

Fig. 2.

In fact, in this case the fluid velocity and the intensity of the magnetic field on the plane $y = 0$ will have only components in the x-direction, and, it follows, the boundary conditions at points $y = 0$ will correspond to those for an infinitely conducting wall. Equations (1.8) in the case under consideration will take the form

$$\frac{dx}{dt} = -\frac{\Gamma}{4\pi}(1-\zeta^2)\frac{x^2}{(x^2+y^2)\,y}, \qquad \frac{dy}{dt} = \frac{\Gamma}{4\pi}(1-\zeta^2)\frac{y^2}{(x^2+y^2)\,x} \qquad (2.1)$$

$$\zeta = \frac{\Phi}{\Gamma}\left(\ln\frac{\lambda^2x^2y^2}{x^2+y^2}\right)^{-1} \qquad (2.2)$$

It is easy to obtain the integral

$$\frac{1}{x^2}+\frac{1}{y^2} = m^2 = \text{const}$$

Therefore ζ = const. Thus we have

$$\frac{dx}{dt} = -\frac{\Gamma}{4\pi}(1-\zeta^2)\frac{1}{m^2y^3}, \qquad \frac{dy}{dt} = \frac{\Gamma}{4\pi}(1-\zeta^2)\frac{1}{m^2x^3} \qquad (2.3)$$

From Equation (2.3) it is evident that for $\Gamma > 0$, $\zeta < 1$ or for $\Gamma < 0$, $\zeta > 1$, the magneto-vortex pair recedes from the wall and shrinks; for $\Gamma < 0$, $\zeta < 1$ or for $\Gamma > 0$, $\zeta > 1$ the pair approaches the wall, expanding, and for $\Gamma = 0$, $\zeta = 1$ the pair is motionless.

Figure 2 shows the trajectory of the motion of the magneto-vortex pair in the direction of the conducting wall. Assuming that rings behave in a similar fashion, it can be proved that a magneto-vortex ring interacts with an infinitely conducting wall exactly as an ordinary vortex ring interacts with a solid wall [5].

3. Motion of a magneto-vortex pair in the direction of a nonconducting wall. The equations of such a motion may be obtained by investigating the motion of two rings having equal radii, equal but opposite circulations, with one of the rings having magnetic energy, the other being purely vortical having zero current. We will assume that the rings are arranged symmetrically with respect to some plane. Then the normal component of velocity at points of this plane is equal to zero, and the normal component of magnetic intensity is different from zero. These are the conditions that we would have on a stationary solid nonconducting wall. As before, the motion of a magneto-vortex ring will imitate the motion of the corresponding magneto-vortex pair. The system of equations for the motion of such a pair, under the condition that the plane $y = 0$ is nonconducting, has the form

$$\frac{dx}{dt} = -\frac{\Gamma}{4\pi}\frac{x^2}{y\,(x^2+y^2)}, \qquad \frac{dy}{dt} = \frac{\Gamma}{4\pi x}\left[\frac{y^2}{x^2+y^2}-\zeta^2\right], \qquad 2\zeta\ln\lambda x = \Phi \qquad (3.1)$$

The equations have the integral

$$\ln\sqrt{\frac{x^2+y^2}{x^2y^2}} = \frac{\Phi^2}{\ln\lambda x} + c \tag{3.2}$$

Denote

$$U = \frac{x^2+y^2}{x^2y^2} = \frac{1}{x^2} + \frac{1}{y^2}$$

Then

$$\frac{1}{2}\ln u = \frac{\Phi^2}{\ln\lambda x} + c, \qquad u = u_1 \exp\left[2\zeta_0{}^2\left(\frac{\ln\lambda a}{\ln\lambda x} - 1\right)\right]$$

Let

$$\frac{y^2}{x^2+y^2} - \frac{\Phi^2}{4\ln^2\lambda x} = f(x) = \frac{1}{x^2u} - \frac{\zeta_0{}^2\ln^2\lambda a}{\ln^2\lambda x} \tag{3.3}$$

along the integral curve (3.2). Then

$$\frac{df}{dx} = \frac{2}{x^3u}\left(\frac{4\zeta_0{}^2\ln^2\lambda a}{\ln^2\lambda x} - 1\right) + \frac{2\zeta_0{}^2\ln^2\lambda a}{x\ln^3\lambda x} \tag{3.4}$$

We will assume that at the initial instant $x = a$, $y = b$, $\zeta = \zeta_0$, $u = u_1$. Then, from the last equation of (3.1), we have

$$\Phi = 2\zeta_0 \ln\lambda a.$$

The behavior of the function $f(x)$ is shown in Fig. 3. The values of the roots $x_1, \ldots, x_n$ of the function $f(x)$ in the interval $1/\lambda < x < \infty$ and their number are determined by the initial conditions. However, the number of roots is always even, since $f(x) \to -\infty$ for $x \to \infty$ and $f(x) \to 0$ for $x \to \infty$, from the negative side.

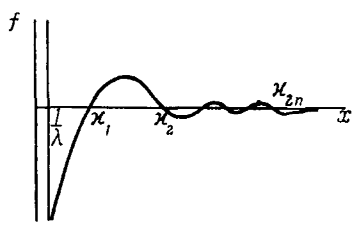

Fig. 3.

Depending on the magnitude of the initial value $x = a$, the following cases may occur:

First case. $a < x_1$. For $\Gamma > 0$ the radius of the ring decreases and the ring approaches the wall with increasing velocity. For $\Gamma < 0$ the radius of the ring grows and the ring recedes from the wall. Depending on the relation between the initial values of magnetic and kinetic energies ζ_0 the following may occur:

(a) For a large value of ζ_0 the root x_1 is large and the ring can recede so far that the presence of the wall is no longer felt; the derivative $dx/dt \to 0$ with increasing y and $x \to x_0 < x_1$. In Fig. 4 are shown the integral curves corresponding to the initial values $\lambda = 5$, $a = 2$, $b = 3$, $\zeta_0 = \sqrt{2}$ (curve 1) and also $\zeta_0 = \sqrt{3}$ (curve 2). For positive value of the circulation Γ the ring approaches the wall very fast, and for negative values it recedes. If there is an aperture in the wall, the ring jumps through it and goes to infinity.

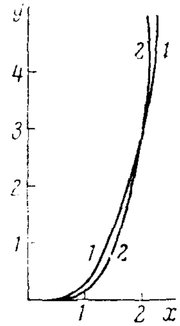

Fig. 4.

(b) If the value ζ_0 is not large (of order 1), the ring will recede from the wall, growing until the time when the value $x = x_1$ is reached; then the velocity dy/dt changes sign and the ring begins to approach the wall, continuing to grow. Then, when the value of x becomes equal to x_2 the ring again begins to recede. Such oscillatory motion will continue until the ring grows so much that the velocity dy/dt becomes zero. Actually, the ring becomes diffuse at some distance from the wall.

Second case. $x_1 < a < x_2$. For $\Gamma > 0$ the diameter of the ring decreases, but the ring moves away from the wall.

(a) If for this case the quantity ζ_0 is small, the ring recedes from the wall.

(b) If ζ_0 is not very small then with decreasing radius the ratio of the magnetic to kinetic energies $\zeta^2 = M/K$ increases and after x reaches the value x_1 the ring returns to the wall. On Fig. 5 are shown the integral curves for the case where ζ_0 is small ($\zeta_0 = 1/2$, $1/\sqrt{3}$). For positive circulation (increasing x) the ring at first recedes from the wall like a pure vortex ring. On Fig. 6 is shown the integral curve corresponding to small difference between the initial values of magnetic and kinetic energies. The ring at first recedes from the wall and then approaches it.

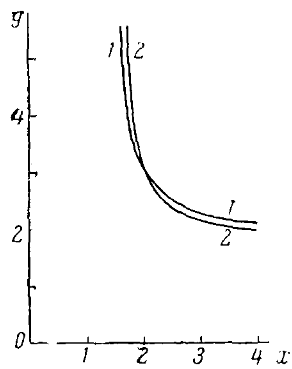

Fig. 5.

For $\Gamma < 0$ the ring expands and approaches the wall; for small values of ζ_0 the root x_2 is located far away, and when x reaches the value x_2 the velocity dy/dt, even though it changes sign, will be so small that the ring will practically stop and diffuse at some distance from the wall (Fig. 5).

If ζ_0 is equal to unity, the root x_0 and the following ones may be relatively near. The ring will exhibit an oscillatory motion toward the wall and away until it also diffuses at some distance from the wall.

Such a regime of the motion is shown in Fig. 6. It corresponds to the initial values $\zeta_0 = 2/\sqrt{5}$, $a = 2$, $b = 4$, $\lambda = 5$. Further cases of the magnitude of the initial values $x = a$ will not differ qualitatively from the case considered in the dependence on the roots of $f(x)$. The difference will consist only of the number of oscillatory movements toward the wall and back. In [1] it was shown that for sufficiently high values of ζ_0 there may be pressures inside the magneto-vortex ring which greatly exceed the pressure in the surrounding gas. It was shown there that the magneto-vortex ring is stable to small perturbations of its surface. These two circumstances make it valid to investigate the magneto-vortex ring as a possible model of ball lightning. The magnetc-vortex ring motion toward a nonconducting wall investigated in this article shows that ball lightning can penetrate into a room through narrow apertures [2].

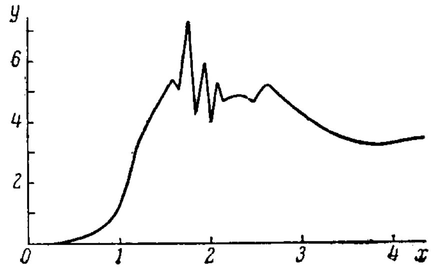

Fig. 6.

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